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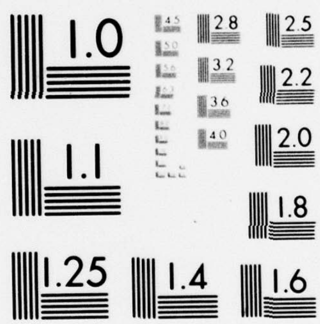

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TRANSFORMS AND APPROXIMATIONS IN COST AND PRODUCTION FUNCTION RELATIONS

by

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Abstract

Process analysis and related approaches to the study of energy economics have made extensive use of Shephard's lemma as well as other aspects of the Shephard-Samuelson transformation theories. A major problem is shown to be present in the use of these transforms to go from cost functions to production possibility sets in that the latter will always be unbounded above. Capacity conditions, which are especially important in energy policy studies, are therefore not adequately addressed. Troubles also occur in the use of translog approximations because of the functional forms which can result when the Shephard-Samuelson transformations are employed. Nondifferentiability is not the primary difficulty with the translog approximations as is shown with an infinitely differentiable function. Relations between other parts of mathematical transform theory, e.g., as exhibited in Laplace transforms, are also indicated along with possible extensions that might be made in the Shephard-Samuelson "duality" theories.

1. Background

The concept of a production function -- as well as most other concepts in micro-theory -- involves an optimizing principle which implies the presence of zero "waste." This is a situation in which no output can be increased at zero cost. The costs to be considered are real costs, of course, which means that waste is present when some output can be obtained without augmenting any input or, conversely, when the already attained outputs can be secured with a release of at least some inputs.

Allowance needs to be made for possible departures from these production function concepts, when, as in the case of energy, large price increases occur over relatively short time periods. Important issues that require attention may otherwise be concealed from view and related parts of micro theory may not be properly applied.1/

1/ See the discussion in [1] concerning waste, chance and mistakes in managerial judgment.

Two routes for effecting such allowances may be outlined as follows:

One might determine whether subsets of firms, or other decision-making units2/, have already effected adjustments to a new frontier. As described in [4], one could then

2/ E.g., not-for-profit organizations.

use the observations from this subset to estimate the amount of waste before effecting adjustments to obtain the production frontiers for the remaining entitites, or at least those entities engaged in "similar" activities.3/

3/ I.e., firms using the same inputs and outputs, See [6]

Alternatively, one might impute some parts of these differences to time-dependent adjustment paths being followed by different firms. For this, one would need explicitly formulated models of production behavior along with optimizing principles for establishing the relevant time paths of adjustment. Then one could again determine the amounts of waste and other types of adjustment that would be required to reach a new equilibrium, and so on. See [4]

These are not the only possibilities. In a recent issue of this journal, for instance, J. M Griffin has combined a "process analysis" model[1/] with an

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1/ This term derives from the title of the work by Markowitz and Manne in [13]

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econometric approach to estimate elasticities and related economic magnitudes in a study of electric power generation.[2/] Using "engineering" data from a variety

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2/ See [10]

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of sources Griffin uses this process analysis approach to generate long-run cost-minimizing inputs corresponding to alternative relative price vectors. This results in a set of "pseudo data."[2/] Via the indicated minimization a

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3/ This term is credited to L. R. Klein, on p. 113 of [10]

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series of points are obtained on a well defined production surface and these in turn are used to estimate a translog cost function. Extensive use of Shephard's lemma and like devices from mathematical ecomics are employed[1] and, because the analysis is long-run, it is assumed that optimal

[1]/See Shephard [15] and [16] and also Samuelson [14].

capacity configurations are selected.[2]

[2]/Griffin [10] p. 114.

Griffin's approach has considerable appeal in its own right and it also relates to other studies which have also made extensive use of these same kind of capacity assumptions and Shephard-Samuelson duality relations en route to effecting translog approximations to the wanted cost and/or production surfaces.[1] It is therefore important to explicitly consider possible diffi-

[2]/See, e.g., E. A. Hudson and D. W.. Jorgenson [11].

culties that can arise from some of the underlying assumptions in these approaches.

To bring one of these assumptions into view we quote from Shephard [16], p. 14 as follows:

> "No limitations will be put upon the available amounts of the factors of production, because this implies reference to some particular production unit which confounds the notion of a production function with some implicit economic decisions or production plan, the variety of which is unlimited, preventing a clear, unambiguous and generally applicable definition of the production function."

In other words, an absence of capacity limitations on <u>all</u> of the relevant inputs is assumed so that, at a minimum, one ought to explore the consequence of relaxing this assumption.

Similarly strong assumptions are made about price-quantity interactions for, as Shephard also notes,

> "Throughout all of these duality theories..., the single most important assumption made limiting their usefulness is that prices ... for input and output vectors are independent of their magnitudes...."1/

1/ The quotation refers to the duality theories that are discussed in the references under [12].

Hopefully one might relax these very severe assumptions since capacity problems occupy positions of central importance in many aspects of energy planning and should therefore not be assumed away or concealed from view. As we shall see, however, the introduction of capacity constraints causes difficulties and may conceal important capacity limits (known to be present) when transforming from production to cost analytic approaches. These limitations can also cause approximation difficulties even for functions with the assumed generality of translog functions. Indeed, as we shall see, when the parameters of such a function are adjusted to provide a good approximation in one region then, under the circumstances we shall be considering, they necessarily yield a bad approximation in other regions.

In the above demonstration of these approximation difficulties we shall use the same quadratic formulations as Griffin.2/ However, we also want to correct an im-

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2/ Hudson-Jorgenson [11], and most other users of the translog function, also use quadratic approximations.

---

pression that the addition of a sufficient number of terms to a translog representation will yield a good fit to any differentiable production (or cost) function. Functions which are infinitely differentiable but which cannot be represented in a translog approximation may be needed to resolve some of the problems we shall be considering. Hence we shall provide an example of this kind of function. Then we shall conclude by trying to point up what our analysis has shown and also indicate possible lines of research that might be followed to resolve these shortcomings without retreating from advances that have been made in energy modeling and analysis.

2. Transform Relations

We can obtain clarification on what is involved if we first turn to other parts of mathematics where similar difficulties are known to occur. For this we turn to transform theory (e.g., Fourier and Laplace transforms) as found in engineering and physics.

For reasons of simplicity let us consider only the Laplace transform. First we recall that the Laplace transform F(p) of f(t) is given by

$$(1) \qquad F(p) = \int_0^\infty e^{-pt} f(t)\, dt$$

for suitable classes of functions.[1/] There also exist various inversion transforms yielding f(t) from F(p), e.g.,[2/]

$$(2) \qquad f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{tp} F(p)\, dp.$$

For cost and production theory there are the elegant relations, which we here propose to refer to as "Shephard transforms," [3/] as follows

$$(3) \qquad C(y,p) = \min.\ p^T x \quad \text{for} \quad \{x:\Psi(y,x)\geq 1\}$$

and

$$(4) \qquad \Psi(y,x) = \min.\ p^T x \quad \text{for} \quad \{p:C(y,p)\geq 1\}.$$

1/ Cf, e.g., D. V. Widder [18].

2/ Again cf. Widder [18] for details.

3/ Cf. Fenchel [9] for geometric characterizations of this type of transform which might also be called "Shephard-Samuelson" transforms in view of their virtual simultaneous publication in [14] and [15].

Herein the set $\{x:\Psi(y,x)\geq 1\}$ is the production possibility set;[1] i.e., it is an alternative characterization of the point-to-set production correspondence $y \rightarrow L(y)$ between the output vector y and the set $L(y) \equiv \{x:$ at least the output vector y is produced $\}$. This set is required to be non-empty and convex and, indeed, to be a cone, possibly truncated below,[2] so that the components of the input vector $x\geq 0$ are always available in the requisite amounts for any $y\geq 0$ that may be stipulated. Similar remarks apply to the price vector p which, in transposed form, is used to generate the cost function $C(y,p)$ specified in (3) as well as the "distance function" $\Psi(y,x)$ specified in (4).

Equivalently we can obtain $C(y,p)$ from the characterization

(5) $\quad C(y,p) = \min.\ p^T x \quad \text{for } x\varepsilon L(y).$

In pseudo-data methods, as in [10], however, we directly determine $C(y,p)$ by estimating parameters in a posited functional form.

Clearly, the relationships between $C(y,p)$ and $\Psi(y,x)$, as exhibited above, are similar to those of $f(t)$ and $F(p)$. One is the transform of the other in each pair. Moreover, with either of these pairs one may start with one of the functions to obtain the other. Each pair is related by the indicated operator. Integration is the operation which effects the transformations in the Laplace pair and minimization is the operator for the Shephard pair. Finally, under suitable conditions there is a bi-unique correspondence between the function and its transform in each pair.

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[1] We are here conforming to the notation of Diewert et al. as in [12].

[2] See Diewert [12.1].

One of the well known relations, so-called "Tauberian Theorems." cf. [18], p. 192, Theorem 4.3, between F(p) and f(t) implies that if $F(p) \sim p^{-\nu}$, $\nu>0$ as $p \rightarrow \infty$ then $t^{\nu-1} / \Gamma(\nu)$ is asymptotic to f(t) as $t \rightarrow 0$. Thus if $\hat{F}(p)$ is asymptotic to F(p) in a neighborhood of ∞, it can generate an $\hat{f}(t)$ also asymptotic to f(t) as $t \rightarrow 0$. However, the inverse transform of $\hat{F}(p)$ may be nowhere close to f(t) for t substantially different from zero, and, a fortiori, there is no reason why $\hat{F}(p)$ need be close to F(p) away from the region of $p = \infty$.

For example, $f(t) = \begin{cases} \sin t, & t \geq 0 \\ 0, & t<0 \end{cases}$ has Laplace transform $F(p) = (p^2+1)^{-1} = \sum_n (-1)^{n+1} p^{-2n}$. Thus, $\hat{F}(p) = p^{-2} - p^{-4}$ (approximates) is asymptotic to F(p) at $p = \infty$ (to the second order). Since $\Gamma(2) = 1$, $\hat{f}(t) = t - t^3/(3!)$ which is asymptotic to sin t as $t \rightarrow 0$. Clearly, $\hat{f}(t)$ is nothing like sin t elsewhere, and this behavior provides a clue to the likely existence of similar phenomena with the Shephard transforms. In other words, the fact of a good approximation in one region need not imply that the fit is good in other regions as well.

A Process Analysis Example

We now proceed to investigate such possiblities by means of linear programming formulations of "process analysis" models. First we note that a system so represented need not have constant returns to scale since, e.g., one may be employing piecewise linear convex functions in the constraints and functionals.1/ Further, in such a formulation the input price may well vary with the level of the associated activities. Still further, the linear program may be employed algorithmically, e.g., as in integer programming, to solve quite arbitrary nonlinear problems.2/ Hence an approach via process analysis need not be limited by the assumption of constant returns to scale, as Griffin assumes, but there are problems associated with the use of the Shephard-Samuelson transforms and other aspects of the analysis that Griffin (like Hudson & Jorgenson [11]) employs.

1/See, e.g., [3]

2/See the discussion of "algorithmic completion of a model" in [3].

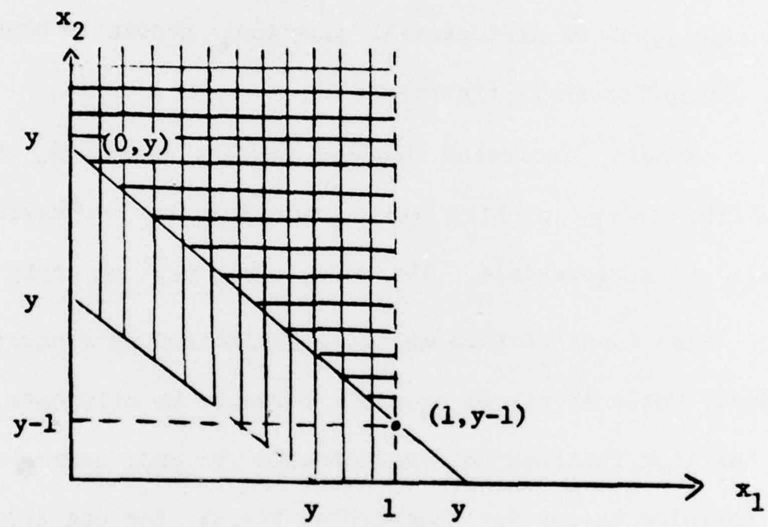
As we have already observed, it is assumed in the Shephard transforms (between production functions and cost functions) that the input prices are constant and independent of the input quantities. This limitation of scope is similar to one also encountered in Laplace transform usages. For instance, the latter are useful for dealing with linear differential equations with constant coefficients even to the extent of providing a general theory and approach to the solution of such equations. Other, more recondite, types of differential equations generally have no such transform theory for their treatment.

Models, including process models, which try to reflect real aspects of the energy problem will generally involve restrictions such that not all y are possible. The set $L(y)$ may be empty for some output vectors y. Some input vectors may also be limited by capacities or other stipulations. While it may be possible formally to attribute infinite values to the cost function for unattainable y and, correspondingly, to attribute infinite values for Shephard's $\Psi(y,x)$ for unattainable values of x — see comments in our concluding section -- no one seems to have dealt explicitly with the discontinuities involved. It should be clear, however, that smooth, regular "everywhere defined" functions must fail to provide adequate approximations in many cases of great importance.

In order to make the preceding considerations more concrete we now proceed by means of a very simple example involving only one output and two inputs. Thus, let

$$(6) \qquad L(y) = \{(x_1,x_2): \; x_1+x_2 \geq y; \; x_1 \leq 1; \; x_1, x_2 \geq 0\}.$$

The set L(y) is shown in the following Figure.[1/] The horizontal shadings represent L(y) for $y>1$. The vertical shadings represent L(y) for $y\leq 1$. Notice that in both cases there are only two "cost efficient" extreme points in the set. Thus in the determination of $C(y,p) = \min . p^T x,\ x \epsilon L(y)$, where $y \geq 0$, one or the other of the extreme points gives the minimum, depending on the relative values of p_1 and p_2, the components of p associated with x_1 and x_2, respectively.



By virtue of what has just been said we can represent the values of C(y,p) in a simple two-way table as follows:

(7) C(y,p) :

	$p_1 \leq p_2$	$p_1 > p_2$
$y \leq 1$	$p_1 y$	$p_2 y$
$y > 1$	$p_1 + p_2(y-1)$	$p_2 y$

1/ Notice that L(y) is not a truncated (below) cone.

Note that in the general case of a polyhedral set of production possibilities involving n input variables x_j and m output variables y_i we could, in principle, construct a larger table with one column for each "cost efficient" production possibility and with one row for each facet in the y_i for which different extreme points could be designated. At any rate, as this example already makes clear, there can be different functional forms for $C(y,p)$ corresponding to the various (y,p) possibilities.

We now develop additional tables of the same type to exhibit some other relevant properties. For instance, taking natural logarithms in the above table yields

(8) $\ln C$:

$\ln p_1+\ln y$	$\ln p_2+\ln y$
$\ln[p_1+p_2(y-1)]$	$\ln p_2+\ln y$

Next, the partial elasticities with respect to input prices are respectively exhibited via

(9)

$\frac{\partial \ln C}{\partial \ln p_1}$:

1	0
$\frac{1}{p_1+p_2(y-1)}$	0

$\frac{\partial \ln C}{\partial \ln p_2}$:

0	1
$\frac{y-1}{p_1+p_2(y-1)}$	1

Finally, we record the efficient input demands x_1^*, x_2^*, which, according to Shephard's lemma, are given by the partial derivatives

$\frac{\partial C}{\partial p_1}$, $\frac{\partial C}{\partial p_2}$:

$$x_1^* = \frac{\partial C}{\partial p_1} \quad :$$

y	0
1	0

(10)

$$x_2^* = \frac{\partial C}{\partial p_2} \quad :$$

0	y
y-1	y

3. Inverse Transforms

Having obtained $C(y,p)$ and its related economic magnitudes from (3) we now proceed to try to obtain $\Psi(y,x)$ by the inverse operations defined by (4) which we now rewrite as

$$\text{(11)} \qquad \Psi(y,x) = \text{inf. } p^T x \text{ for } p \geq 0 \text{ and } C(y,p) \geq 1.$$

For $C(y,p)$ as in (7), this inverse operation can be formulated in terms of linear programming problems for which we need only examine and compare the values of $p^T x$ at extreme points.

For example, for $y \leq 1$, $x>0$ and $p_1 \leq p_2$, we have from (7) that $p_1 y \geq 1$. Hence for a minimum $p_1 y = 1$, or $p_1 = 1/y$. Also $p_1 \leq p_2$ implies for a minimum that $p_2 = p_1$. Therefore, the minimum of $p_1 x_1 + p_2 x_2$ for $p_1 \leq p_2$ is given by $(x_1+x_2)/y$. If instead $p_1 > p_2$, this same argument interchanges p_1 and p_2 and we get the same result -- i.e., $(x_1+x_2)/y$. Thus for $y \leq 1$, $\Psi(y,x) = (x_1+x_2)/y$ *no matter which price regime applies.*

The reasoning becomes more involved for $y>1$, but reduces to consideration of only two cases, namely,

$$\text{(12)} \qquad x_2 \leq x_1(y-1) \text{ and } x_2 > x_1(y-1).$$

Finally, we obtain for $\Psi(y,x)$ the piecewise representation,

(13) $\Psi(y,x)$:

	$x_2 \leq x_1(y-1)$*	$x_2 > x_1(y-1)$*
$y \leq 1$	$(x_1+x_2)/y$	$(x_1+x_2)/y$
$y>1$	$x_2/(y-1)$	$(x_1+x_2)/y$

*Applicable only to cells in second row.

It may be noted that the production possibility set for any fixed y given by $\Psi(y,x) \geq 1$ is a truncated (below) cone in contrast to L(y), which is not. However, the so-called efficiency frontier is the same for these two <u>different</u> production possibility sets.

Thus, if the production possibility sets are derived from C(y,p) as $\{(y,x): \Psi(y,x) \geq 1, x,y \geq 0\}$ they will always be truncated (below) cones and will fail to give information about limitations of the production possibility sets. Such limitations may be of critical importance for policy issues -- such as are now being examined by means of this transform theory -- in areas like energy costs, prices and substition possibilities.

The point is that the production possibility set developed for any cost function by the Shephard-Samuelson transforms is necessarily unbounded above. This occurs no matter how the cost function is developed -- by pseudo data or otherwise.[1/] Limitational possiblities on various inputs actually ought to be brought to the fore for explicit evaluation since they represent a central feature of the energy problem for many types of policy decisions. Note that such limitational possibilities may not be immediately apparent since their effects may be experienced in terms of complex interactions between the various inputs. Hence any approach which fails to bring such possibilities to the fore is likely to be seriously deficient as a guide to contemporary problems of energy policy.

[1/] E.g., Hudson-Jorgenson [11] as well as Griffin [10] both proceed via the cost function and hence both are comprehended in these comments despite other differences in their approaches.

4. Translog Representations

The above development indicates some of the difficulties in utilizing the Shephard transforms in these process analysis approaches. Now we delineate other problems that can arise when employing translog approximations to the resulting data when input limitations are present. Thus suppose we approximate C(y,p) to the second order in the logarithms of p as in a translog function. For simplicity of notation, we do not here bother with all of the corresponding approximations in y since taking derivatives with respect to p_1 and p_2 will remove the terms involving only y. Therefore, for this translog function approximation, we write.

$$\begin{aligned} \ln\tilde{C} &= a_{10}\ln p_1 + a_{02}\ln p_2 + a_{11}(\ln p_1)^2 \\ &+ a_{22}(\ln p_2)^2 + a_{12}\ln p_1 \ln p_2 + a_{13}\ln p_1 \ln y + a_{23}\ln p_2 \ln y \\ &+ \text{terms in logarithms of } y. \end{aligned} \tag{14}$$

Thus we obtain

$$\frac{\partial \ln\tilde{C}}{\partial \ln p_1} = a_{10} + 2a_{11}\ln p_1 + a_{12}\ln p_2 + a_{13}\ln y \tag{15}$$

which we can compare to the first expession in (9) -- viz.,

$\frac{\partial \ln C}{\partial \ln p_1}$:

1	0
$\frac{1}{p_1 + p_2(y-1)}$	0

and

$$\text{(16)} \qquad \frac{\partial \ln \tilde{C}}{\partial \ln p_2} = a_{02} + 2a_{22} \ln p_2 + a_{12} \ln p_1 + a_{23} \ln y$$

which we can compare to the second expression in (9) -- viz.,

$\frac{\partial \ln C}{\partial \ln p_2}$:

0	1
$\frac{y-1}{p_1+p_2(y-1)}$	1

There is no way of choosing the values of a_{ij} to get a good match in more than two regions for either one of these two elasticities. E.g., suppose we approximate $\frac{\partial \ln C}{\partial \ln p_1}$ by $\frac{\partial \ln \tilde{C}}{\partial \ln p_1}$ in the neighborhood of $p_1=p_2=1$ and $y=1$, where $\ln p_1=\ln p_2=0$ and $\ln y=0$. Then for a good approximation in the neighborhood of $p_1<p_2$, $y\leq 1$ we must have $a_{10}\approx 1$. But in the neighborhood of $y<1$, $p_1<p_2$ we need $a_{10}\approx 0$. Both values cannot be simultaneously assigned to a_{10}. Similar remarks apply to the other elasticity and to other neighborhoods.

Finally, we turn to the efficient inputs obtained from Shephard's lemma for further comparisons via

$$\text{(17)} \qquad \tilde{x}_1^* = \frac{\partial \tilde{C}}{\partial p_1} = \frac{\tilde{C}}{p_1}\frac{\partial \ln \tilde{C}}{\partial \ln p_1} = \frac{\tilde{C}}{p_1}\left[a_{10} + 2a_{11}\ln p_1 + a_{12}\ln p_2 + a_{13}\ln y\right]$$

instead of the first expression in (10) -- i.e.,

$x_1^* =$

y	0
1	0

Similarly,

$$(18) \qquad \tilde{x}_2^* = \frac{\partial C}{\partial p_2} = \frac{\tilde{C}}{p_2}\frac{\partial \ln\tilde{C}}{\partial \ln p_2} = \frac{\tilde{C}}{p_2}\left[a_{02} + 2a_{22}\ln p_2 + a_{12}\ln p_1 + a_{23}\ln y\right]$$

instead of the second expression -- i.e.,

$$x_2^* = \begin{array}{|c|c|}\hline 0 & y \\ \hline y-1 & y \\ \hline \end{array}$$

If, as before, we focus on approximation in the neighborhood of $y=1$, $p_1=p_2=1$ (hence $\ln p_1=\ln p_2=0=\ln y$) then $\tilde{C}/p_1 \approx \tilde{C}/p_2 \approx 1$. Thus in (17) and (18) the values of a_{10} and a_{02} play the same roles as before with analogous consequences and evidently similar conclusions as in the preceding case again apply.

In this case the source of the difficulty in the wanted approximation is known. In other cases, and especially when statistical noise is present, it may be concealed from view. Moreover, in longer run studies such as those essayed by Griffin, Hudson-Jorgenson <u>et al</u>., the elasticities and other relevant economic quantities may be wide of the mark in the future — e.g., because of a change in relative prices -- precisely because they represent good fits to present data (including pseudo data).

The above very simple examples thus provide what is wanted. These difficulties are not matters which can be handled by simply adding terms as in a power series representation. The failure is in the functional forms <u>per se</u> and hence is a failure of the methodology itself.

To the argument that the translog function is intended as a good approximation to sufficiently differentiable functions (which are otherwise arbitrary) we now enter two remarks as follows. First, as our simple example shows, process analysis models will generally yield only piecewise differentiable functions. Second, a local second order approximation by a translog function to a differentiable function may be unsatisfactory even when the latter is infinitely differentiable. [1/] A simple example involving only one variable is as follows

$$\ln C = a_{00} + a_{10}\ln p_1 + a_{11}(\ln p_1)^2 + f(p_1),$$

(1) where

$$f(p_1) = \begin{cases} 0 & , 0 \leq p_1 \leq 1 \\ e^{-(p_1-1)^{-2}} \; e^{p_1^4} & , p_1 > 1. \end{cases}$$

Thus, in the neighborhood of $p_1=1$, lnC is closely approximated by $a_{00} + a_{10}\ln p_1 + a_{11}(\ln p_1)^2$. As p_1 increases beyond 1, however, $f(p_1)$ increases as the p_1^4 power of e and cannot possibly be overtaken by logarithmic terms. [2/] Such functional forms may be needed to reflect the effect of capacity constraints on prices as limits on the underlying inputs are approached. (See concluding remarks in the next section.) Evidently the translog approximations could not be satisfactorily employed for this purpose.

1/ In their original publication [7] Professors Christensen, Jorgenson and Lau assert that the translog function provides a second order approximation to an "arbitrary" functional form but their subsequent usage makes it clear that they are restricting themselves to differentiable functions with a sufficient number of derivatives. See, e.g., [8].

2/ Other limitations that may be present in the translog and other "flexible" functional forms are examined in [2] and [17].

5. Conclusion

We have shown that constraint limitations (possibly present in implicit form) that ought to be given explicit attention in empirical studies of energy policies are, in fact, impossible to obtain by Shephard-Samuelson transforms from the cost functions. Where production possibility sets are acutally limited above, even "flexible" functional forms such as the translog cannot approximate the correct cost function except locally in regions where the capacity limitations are not binding. In fact, attempts at good fits in one of these regions then necessarily results in poor fits in others.

As exemplified by even the simpler Laplace transform discussed in Section 2, the transform of an approximating function may be far from the transform of the exact function. One should therefore always be aware of the possibility of this kind of behavior when applying any transform method.

The Shephard-Samuelson "duality" theory is dependent on prices being independent of volume and mix. This assumption is also not likely to be satisfied in either short-range or long-range energy policy problems. It may be possible to extend this theory to cover at least some cases in which prices are dependent on volumes. If one can approximate capacity limitations by extremely high prices for production at or above these limits, for example, then a resolution of the production limitation difficulties might also be secured in that impossible levels of production might be signalled by "steeply rising" prices. These are important topics for research which would be of value in properly extending the important advances that Griffin, Hudson-Jorgensen et al. have already effected for the modeling problems in areas such as energy policy. Until this research has progressed sufficiently far, however, it is important to bring these limitations very explicitly to the fore in order to avoid possible pitfalls for many priority energy policy decisions that are presently being considered by means of these methods from mathematical economics with their accompanying (very strong) underlying assumptions.

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13. ABSTRACT

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Process analysis and related approaches to the study of energy economics have made extensive use of Shephard's lemma as well as other aspects of the Shephard-Samuelson transformation theories. A major problem is shown to be present in the use of these transforms to go from cost functions to production possibility sets in that the latter will always be unbounded above. Capacity conditions, which are especially important in energy policy studies, are therefore not adequately addressed. Troubles also occur in the use of translog approximations because of the functional forms which can result when the Shephard-Samuelson transformations are employed. Nondifferentiability is not the primary difficulty with the translog approximations as is shown with an infinitely differentiable function. Relations between other parts of mathematical transform theory, e.g. as exhibited in Laplace transforms, are also indicated along with possible extensions that might be made in the Shephard-Samuelson "duality" theories.

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